Using Extreme Value Theory to Optimally Design PV Applications under Climate Change

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***Abstract*-A novel design method for photovoltaic applications (PVA) is proposed based on Extreme Value Theory under climate change. Traditionally, one day or one year's climate data are used in the design of PV applications. However, the fact is that weather conditions in two years won't be the same.Therefore, the PV applications designed by the traditional way may fail in the extreme weather conditions. This paper illustrates a novel method to design a PV application reliable under the climate-change situation.**

Keywords-Climate Change; Extreme Value Theory; PV Application; Sizing; Climate Cycle

I. INTRODUCTION

Design of Photovoltaic applications (PVA) is an active research area in the literature. A very comprehensive review about this topic can be referred to [1]. The technical introduction of PVA can be found in [2]. The traditional methods of power system reliability evaluation are discussed in [3]. Gordon [4] gave an optimal sizing technique based on the stochastic process. Egido and Lorenzo [5] presented an early review and proposed a method to take the advantages from both numerical and analytic approaches. Markvart et al. [6] proposed a sizing technique based on the extreme events.

The design of PV applications mainly involves determining the number of PV plates and the number of batteries in the application. Most of them only use one day or one year's data to do the design task. However, the fact is that the weather conditions in a single site in two years will not be the same. Therefore, the PV applications designed by the traditional way may fail in the extreme weather conditions.

Extreme Value Theory (EVT) [7, 8] is used in the modeling of extreme events occurred in a time series. Since the maximal climate cycle in a year normally cannot be predicted, it is taken as a random event and can be modeled by EVT. EVT has various findings that fit different situations. This paper mainly illustrates the design of PV applications in terms of block maximum. The realistic data from a real site are employed to the illustration. A comprehensive version of this approach can be referred to [9].The remainder of the work is described as follows: The assumptions for illustrating the approach are presented in Section 2. The mathematic preliminaries are addressed in Section 3. The illustration of the proposed method with realistic data is in Section 4. Finally, Section 5 is the conclusion.

II. ASSUMPTIONS

To simplify the illustration, it is assumed to satisfy the following assumptions:

1. The system obeys energy conservation law.
2. The load is constant.
3. The system is optimized by the vendor and the average power of the PV panel is used in calculation.
4. The batteries are initially full.
5. The batteries are only charged by the PV system at day.
6. The load is supplied partly by the PV system at day and totally by the batteries at night.

III. PRELIMINARIES

A. Extreme Value Theory

EVT is a method used for the evaluation of LPP_x under observed data. The extreme value theory gives the following important results [10]:

Theorem 3.1. Suppose X_1, X_2, …, X_n are iid random variables. If there are constants $a_n \in \mathbb{R}$, $b_n > 0$, M_n (the order statistics for X_n) and some non-degenerate limit distribution H such that

$$\lim_{n\to\infty} \Pr\left(\frac{M_n - a_n}{b_n} \le x\right) = H(x)$$

then H is one of the following extreme value distributions:

$$\text{Frechet: } \Phi_\alpha(x) = \begin{cases} 0, & x \le 0 \\ e^{-x^{-\alpha}} & x > 0 \end{cases}, \alpha > 0,$$

$$\text{Weibull : } \Psi_\alpha(x) = \begin{cases} e^{-(-x)^{-\alpha}}, & x \le 0 \\ 0 & x > 0 \end{cases}, \alpha > 0,$$

$$\text{Gumbel : } \Lambda_\alpha(x) = e^{-e^{-x}}, x \in \mathbb{R}$$

A more general form called the generalized extreme value distribution (GEVD) of the three distributions can be expressed as

$$H_{\xi,\mu,\sigma}\left(\frac{x-\mu}{\sigma}\right) = \begin{cases} e^{-(1+\xi\frac{x-\mu}{\sigma})^{-1/\xi}}, & \xi \ne 0, \\ e^{-e^{-\frac{x-u}{\sigma}}}, & \xi = 0, \end{cases}$$

where ξ is the tail index, μ is the position parameter, σ is the scaling parameter.

Theorem 3.1 states that the tail of any iid random variable with unknown distribution will asymptotic converge to the

GEVD. This is a very important result of precisely estimating LPP_x mathematically.

B. Calculation of LPP_x

LPP_x is defined as:

$$LPP_x = \Pr(M_n > x) = 1 - H_{\xi,\mu,\sigma}(\frac{x-\mu}{\sigma}).$$

C. Climate Cycle Creation

The climate cycle can be defined as the specific charge time to charge full the batteries. The following flow chart is proposed to create such climate cycle:

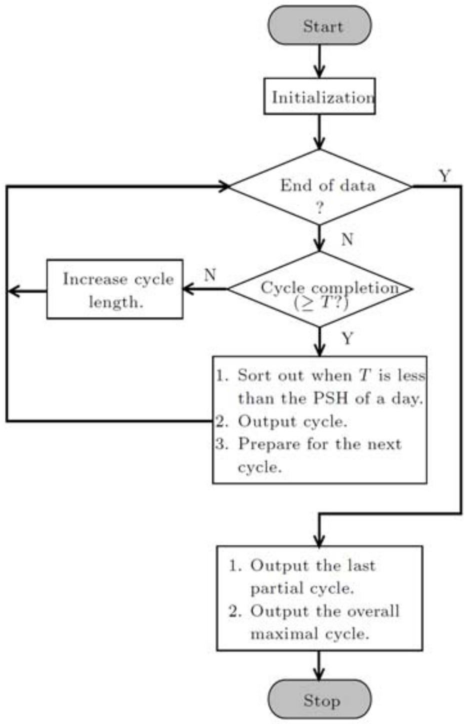

Fig. 1 The flowchart of creating climate cycles

We define a symbol $\hat{\Sigma}$ to do the summation. By definition, a climate cycle c_i may start from the s_i percentage of the PSH at the last day of the previous cycle c_{i-1} and end at the (1 - s_{i+1}) percentage of the PSH at the last day of this cycle. Let Tc_i be the cycle time (i.e., the discharge time) of cycle c_i, then it is expressed as

$$\hat{\Sigma}_{c_i} 24 \equiv Tc_i = \begin{cases} s_i e_{(i-1)_{R'}} + 12 - \dfrac{e_{(i-1)_{R'}}}{2} + 24(R-1) \\ +(1-s_{i+1})e_{i_R} + 12 - \dfrac{e_{i_R}}{2}, \quad 1 < i \le Q, \\ 24(R-1) + (1-s_{i+1})e_{i_R} + 12 - \dfrac{e_{i_R}}{2}, \quad i = 1. \end{cases}$$

In unit of a day, it is denoted as

$$Td_i = \begin{cases} e_{(i-1)_{R'}}(s_i - 1/2)/24 + R + e_{i_R}(1/2 - s_{i+1})/24, & 1 < i \le Q, \\ R - 1/2 + e_{i_R}(1/2 - s_{i+1})/24, & i = 1. \end{cases}$$

Td_i will be used in the later computation of $LPP_{Tdi.}$

Suppose there are Y years. According to the extreme value theory for block maxima, we define the random variables as

$X_j = \max\{Td_i | c_i \text{ is in year } j\}$, $1 \le j \le Y$.

D. Sizing Formulas

Let $Tc_{max} = \max\{Tc_i \mid 1 \le i \le Q\}$. How to choose a suitable charging time T and get the corresponding Tc_{max} depends on the given weather condition. Empirical study shows that the relationship between Tc_{max}/T and T is not linear, actually it is a nearly convex function. Figure 2 shows this relationship.

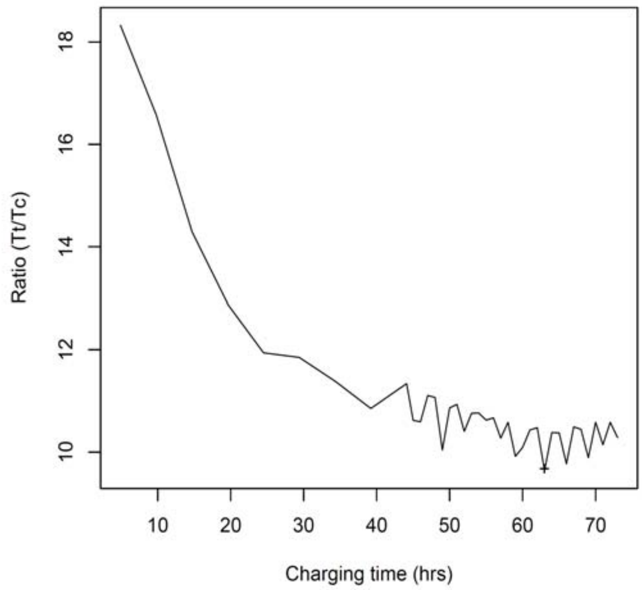

Fig. 2 The convex function realtionship

Two sizing formulas are summarized from the observation.

For PV panels:

$$P_0 = L_0(1 + \frac{1}{RTE}(\frac{Tc^*_{\max}}{T^*} - 1)).$$

For batteries:

$$VI = \frac{L_0}{RTE}(\frac{Tc^*_{\max}}{T^*} - 1).$$

where RTE is round trip efficiency, P_0 is the power of PV panels, VI is the power of batteries.

IV. EXAMPLES

Two sets of the solar radiation data in Florida, USA are illustrated: one is for the in-sample test measuring daily solar radiation for 30 years from 1961 to 1990; the other is for the out-of-sample test measuring daily solar radiation for 15 years from 1991 to 2005. Both data are obtained from the Renewable Resource Data Center of the National Renewable Energy Laboratory, USA. Assume that there is a constant daily load of 4 kWh (48 V) uniformly distributed in a day to be powered by the PV system (205 W, 48 V for one plate) with batteries regulated, where 205W is the average power of the PV panels during PSH. Then, each module of the battery bank needs 4 batteries (lead-acid type, 12 V, 100Ah) cascaded. The RTE of the battery is 0.9.

At first, the empirical analysis of the ratio $Tc_{max}/\ T$ vs. T is conducted. The minimal ratio 9.42851 is at 58.7 h (=T*) of the turn in this analysis. The corresponding length of maximal climate cycle is 23.06058 days $Td\ *_{max}$.

In in-sample test, we got the optimal PV panels are 9 plates:

$$P_0 = L_0\left(1+\frac{1}{RTE}\left(\frac{Tc^*_{max}}{T^*}-1\right)\right) = \frac{4000}{24}\left(1+\frac{1}{0.9}(9.42851-1)\right)$$
$$= 1727.502(\mathrm{W}) = 8.43 \text{ plates}.$$

The optimal batteries are 4 modules:

$$VI = P_0 - L_0 = 1727.502 - \frac{4000}{24} = 1560.835(\mathrm{W}) = 3.25 \text{ modules}.$$

The reliability estimation is 0.02981 which comes from the EVT fitting by MLE method. The fitting parameters are: ξ = 0.000001, μ = 1.048 and σ= 20.947. Figure 3. is the fitted curve.

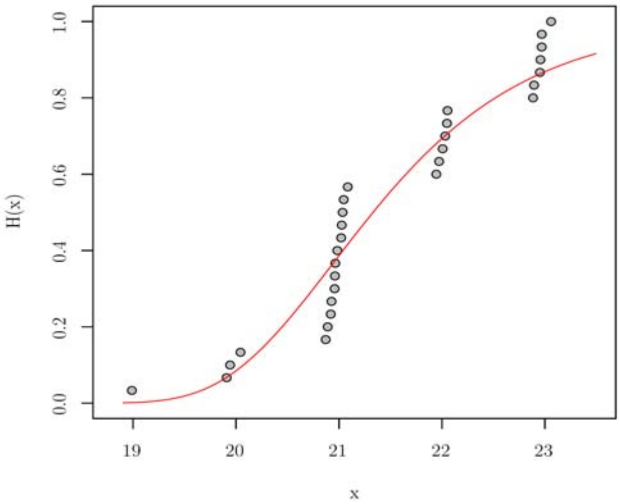

Fig. 3 The fitted extreme climate model

This result is as good as that by the traditional sizing curve method, but with more mathematic insights.

In out-of-sample test, we verify what happened to this design of PV applications. The climate statistics for the next 15 years are in Figure 4.

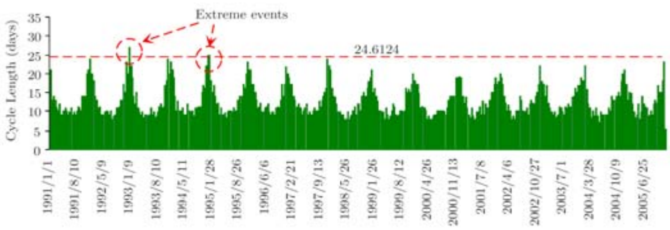

Fig. 4 The extreme events in the next 15 years

The results show that the optimal design is feasible for the future climates. However, three extremes are discovered by this method, which would occur at a frequency of per 317.5611, 48.146 and 47.7783 years respectively. These are the evidences of climate change.

V. CONCLUSIONS

This paper illustrates the way of how to use the modern mathematics in the real applications. A novel design method for PV applications is proposed. Many interesting findings are obtained, which are never explored before. More realistic climate data are encouraged for the future verification of this method.

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REFERENCES

[1] W. Zhou, C. Lou, Z. Li, L. Lu, and H. Yang, “Current status of research on optimum sizing of stand-alone hybrid solar-wind power generation systems,” Appl Energy, vol 87, pp. 380−389, 2010.

[2] S. Silvestre, Review of system design and sizing tools, Oxford: Elsevier, 2003.

[3] R. Billinton, and R. N. Allan, Reliability evaluation of power systems, 2nd ed. New York and London: Plenum Press, 1992.

[4] J. M. Gordon, “Optimal sizing of stand-alone photovoltaic solar power systems,” Sol Cells, vol 20, pp. 295 – 313, 1987.

[5] M. Egido, and E. Lorenzo, “The sizing of stand alone PV-system: a review and a proposed new method,” Sol Energy Mater Sol Cells, vol 26, pp. 51−69, 1992.

[6] T. Markvart, A. Fragaki, and J. N. Ross, “PV system sizing using observed time series of solar radiation,” Sol Energy, vol 80, pp. 46−50, 2006.

[7] E. Castillo, Extreme value theory in engineering. London/San Diego: Academic Press, 1988.

[8] J. Galambos, “Extreme value theory for applications,” In: Galambos J, Lechner J, Simiu E, editors. Proceedings of the conference on extreme value theory and applications. Springer, pp. 1−14, 1993.

[9] S. G. Chen, “An efficient sizing method for a stand-alone PV system in terms of the observed block extremes,” Appl Energy, vol 91, pp. 375−384, 2012.

[10] R. A. Fisher, and L. H. C. Tippett, “Limiting forms of the frequency distribution of the largest or smallest member of a sample,” Math Proc Cambridge Philos Soc, vol 24, pp. 180−90, 1928.

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